Symbolic Homotopy Techniques for Structured Multivariate Polynomial Systems

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Abstract

In the context of solving systems of polynomial equations, homotopy refers to the process of deforming a simple system $G(\boldsymbol{X})$, where $\boldsymbol{X} = (x_1, \ldots, x_n)$ is a sequence of variables, whose solutions are known, into a target system $F(\boldsymbol{X})$ that we aim to solve. There are two main approaches to homotopy methods: numerical and symbolic. While the core mathematical idea, by using the continuous transformation $H(t, \boldsymbol{X}) = (1 - t) \cdot G(\boldsymbol{X}) + t \cdot F(\boldsymbol{X})$, is the same, the way we handle that transformation differs between the symbolic and numerical worlds.

Numerical homotopy methods approximate solutions by tracking solution paths as the parameter t varies from 0 to 1, using predictor-corrector techniques. In contrast, symbolic homotopy methods treat t as an algebraic parameter and study the structure of the ideal generated by the homotopy system in order to obtain a parametric description of the solutions for all values of t simultaneously. Then specilizing this formula at $t = 1$ gives a parametrization for the solution set of the target system $F(\boldsymbol{X})$.

In this talk, we focus on symbolic homotopy techniques for solving multivariate polynomial systems, with a particular emphasis on determinantal systems arising from minors of polynomial matrices, illustrating how exploiting algebraic structure can lead to efficient solution methods.